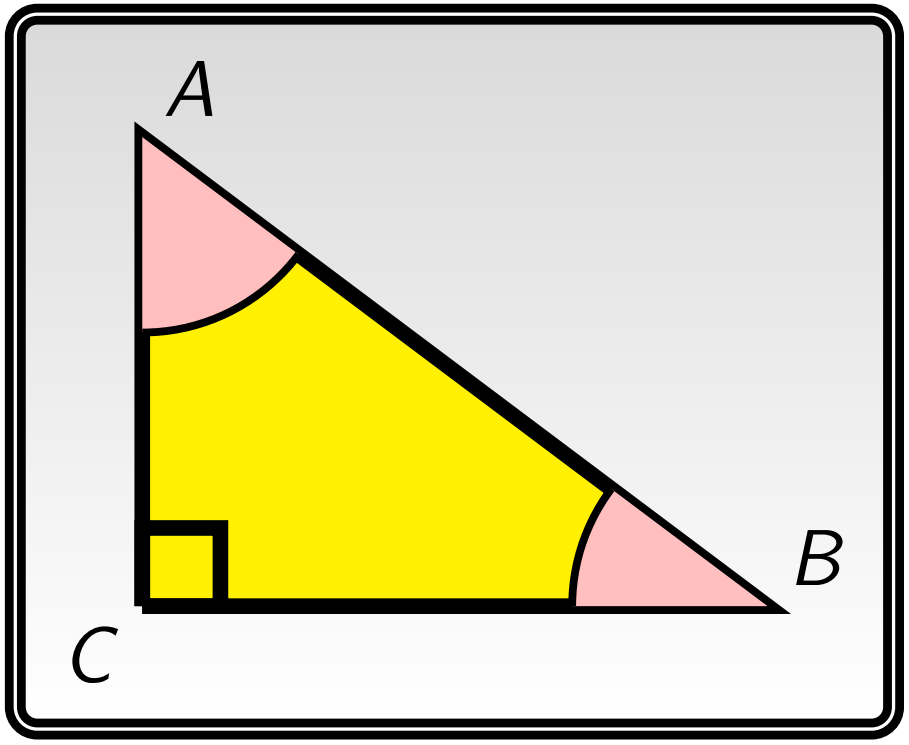
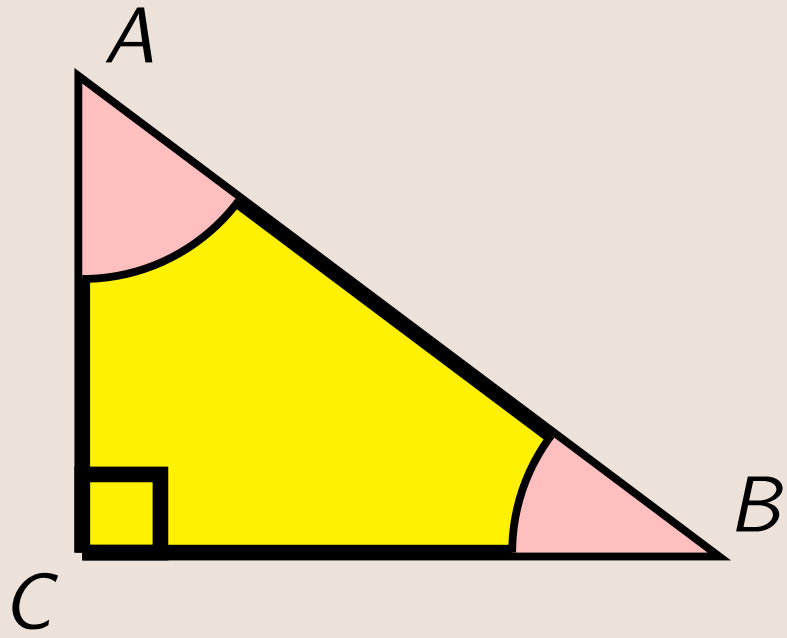
Trigonometry DLA Series



Right Triangle

Angles & Sides

In this DLA, we are are going to study the right triangle.



$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$m\angle C = 90^\circ$$

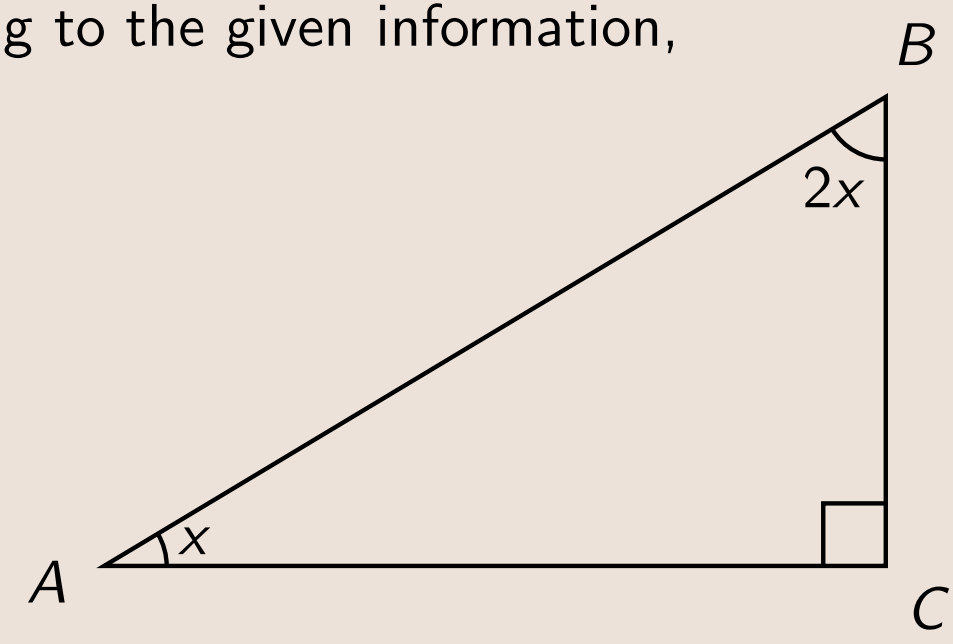
Angles A and B are complementary angles.

Example:

In triangle ABC, angle B is twice angle A, and angle C is 90°. Find the measure of missing angles.

Solution:

Let x be the measure of angle A, draw a triangle, and label each angle according to the given information,



Solution(continued):

$$\boxed{m\angle A} + \boxed{m\angle B} + \boxed{m\angle C} = 180^\circ \qquad \text{(Given Fact)}$$

$$\boxed{x} + \boxed{2x} + \boxed{90} = 180^\circ \qquad \text{(Substitution)}$$

$$3x + 90 = 180 \qquad \text{(Simplify)}$$

$$3x = 90 \qquad \text{(Equation Property)}$$

$$x = 30 \qquad \text{(Division property)}$$

So angle A is 30°, and angle B is 60°.

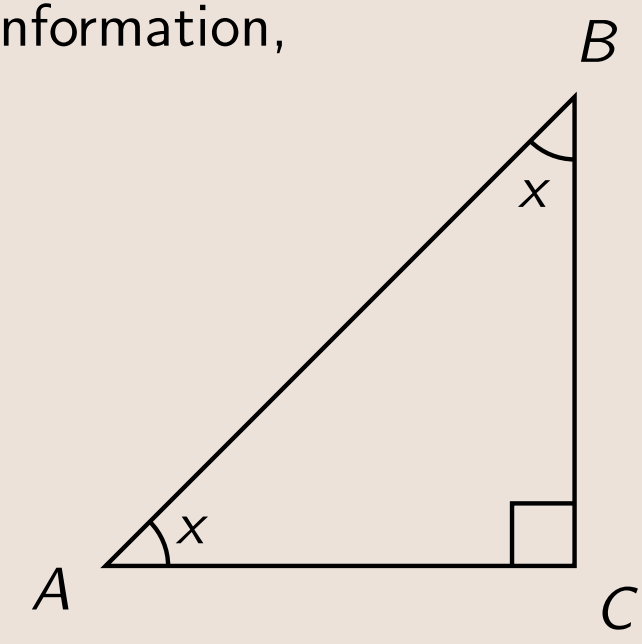
$$\boxed{m\angle A = 30^\circ,\ m\angle B = 60^\circ}$$

Example:

In triangle ABC, angles A and B are equal and angle C is 90°. Find the measure of missing angles.

Solution:

Let x be the measure of angle A, draw a triangle, and label each angle according to the given information,



Solution(continued):

$$\boxed{m\angle A} + \boxed{m\angle B} + \boxed{m\angle C} = 180^\circ \qquad \text{(Given Fact)}$$

$$\boxed{x} + \boxed{x} + \boxed{90} = 180^\circ \qquad \text{(Substitution)}$$

$$2x + 90 = 180 \qquad \text{(Simplify)}$$

$$2x = 90 \qquad \text{(Equation Property)}$$

$$x = 45 \qquad \text{(Division property)}$$

So angle A is 45°, and angle B is 45°.

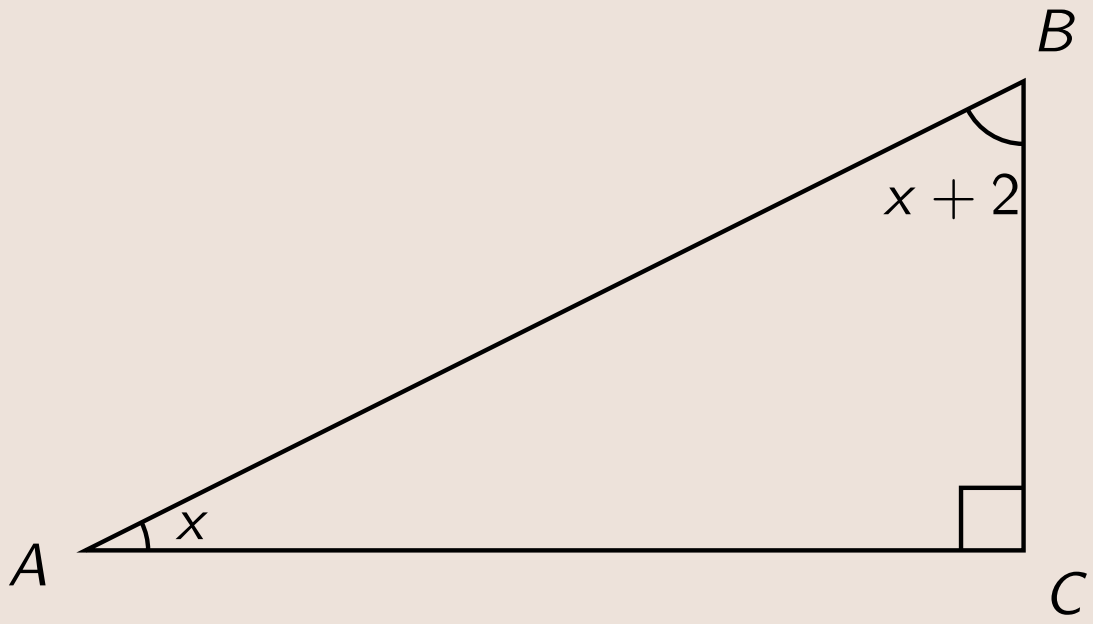
$$\boxed{m\angle A = 45^\circ,\ m\angle B = 45^\circ}$$

Example:

In triangle ABC, the measures of angles A and B are two consecutive even integers and angle C is 90°. Find the measure of missing angles.

Solution:

Let x be the measure of angle A, and angle B would be $x + 2$, then draw a triangle, and label each angle according to the given information,



Solution(continued):

$$\boxed{m\angle A} + \boxed{m\angle B} + \boxed{m\angle C} = 180^\circ \qquad \text{(Given Fact)}$$

$$\boxed{x} + \boxed{x+2} + \boxed{90} = 180^\circ \qquad \text{(Substitution)}$$

$$2x + 92 = 180 \qquad \text{(Simplify)}$$

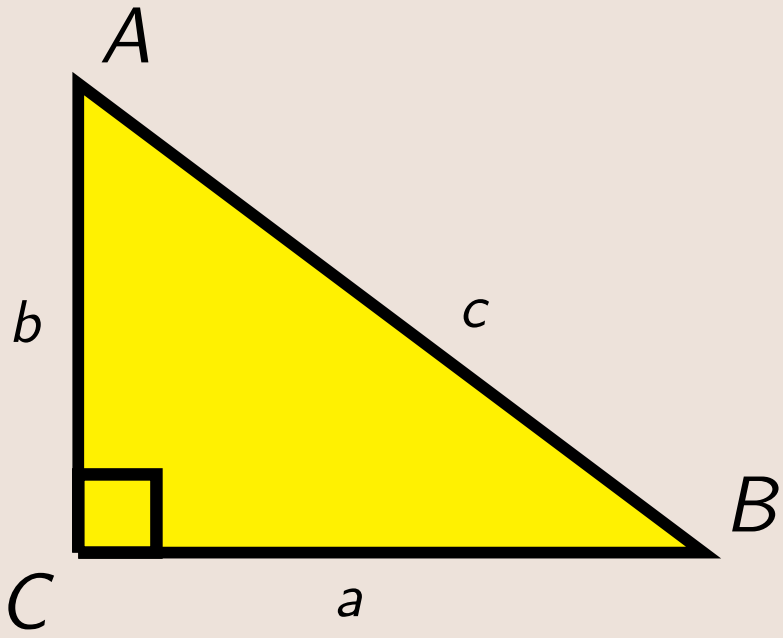
$$2x = 88 \qquad \text{(Equation Property)}$$

$$x = 44 \qquad \text{(Division property)}$$

So angle A is 44°, and angle B is 46°.

$$\boxed{m\angle A = 44^\circ,\ m\angle B = 46^\circ}$$

Now we study the sides of a right triangle.



Sides a and b are called legs.

Side c is called hypotenuse.

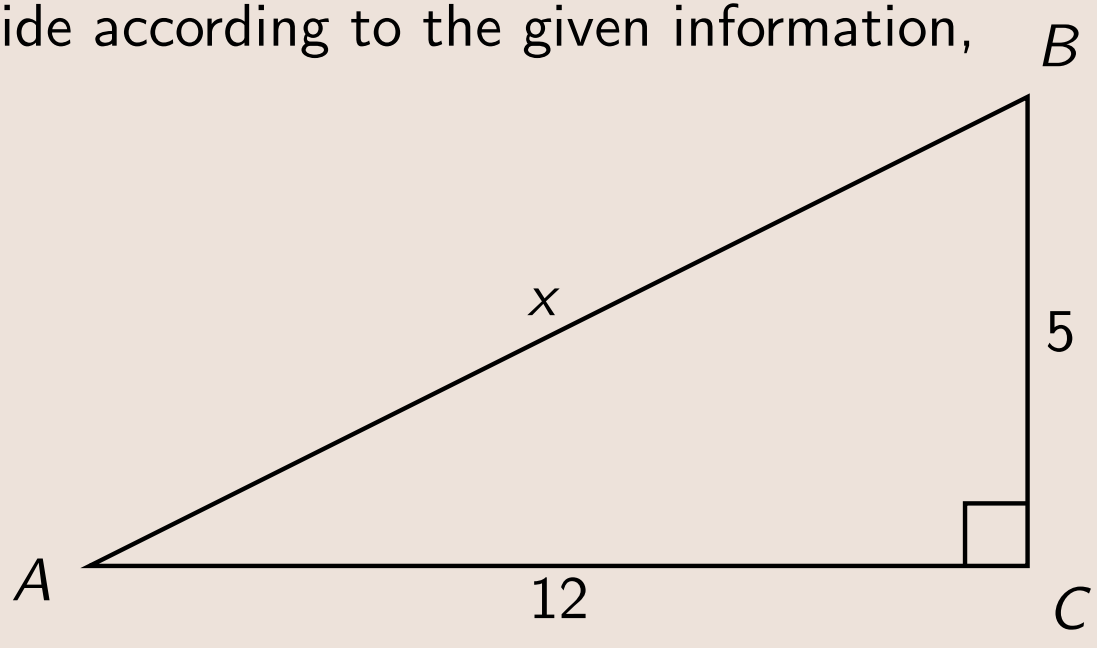
Pythagorean Theorem: $a^2 + b^2 = c^2$

Example:

Two legs of a right triangle are 5 and 12 inches. Find its hypotenuse.

Solution:

Let x be the measure of its hypotenuse, then draw a triangle, and label each side according to the given information,



Solution(continued):

Using the pythagorean theorem,

$$\boxed{a^2} + \boxed{b^2} = \boxed{c^2} \qquad \text{(Given Fact)}$$

$$\boxed{5^2} + \boxed{12^2} = \boxed{x^2} \qquad \text{(Substitution)}$$

$$25 + 144 = x^2 \qquad \text{(Simplify)}$$

$$169 = x^2 \qquad \text{(Simplify)}$$

$$\sqrt{169} = x \qquad \text{(Root property)}$$

$$13 = x \qquad \text{(Root property)}$$

So the hypotenuse is 13 inches.

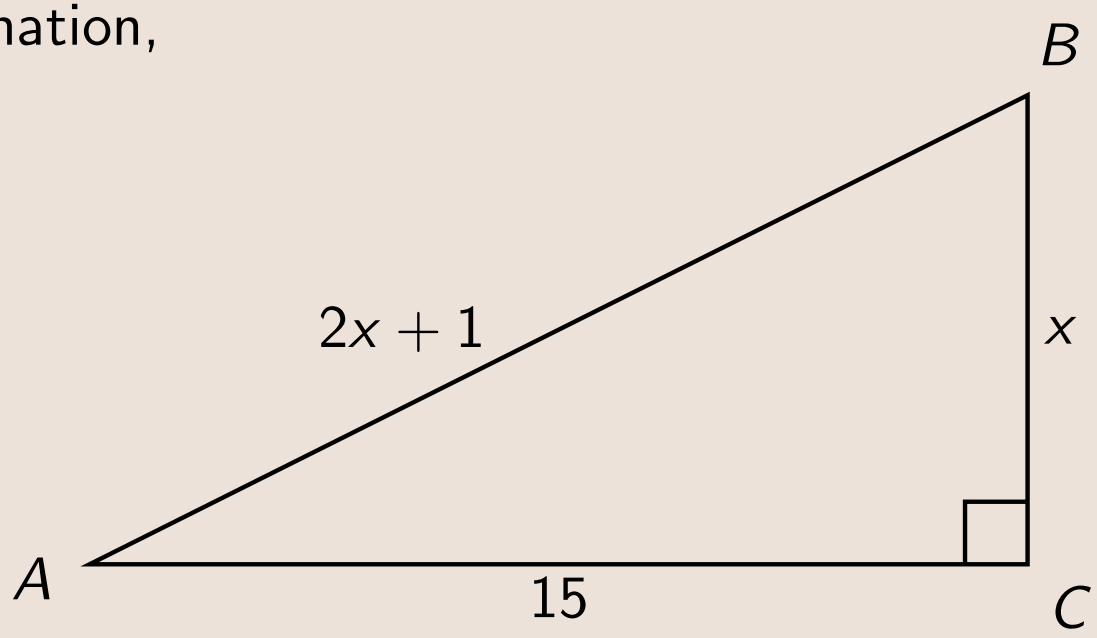
The hypotenuse is 13 inches.

Example:

One leg of a right triangle is 15 cm. Its hypotenuse 1 cm more than twice the other leg. Find the missing side and the hypotenuse.

Solution:

Let x be the measure of the missing leg, then its hypotenuse is $2x + 1$. Draw a right triangle, and label each side according to the given information,



Solution(continued):

Using the pythagorean theorem,

$$\boxed{a^2} + \boxed{b^2} = \boxed{c^2} \qquad \text{(Given Fact)}$$

$$\boxed{15^2} + \boxed{x^2} = \boxed{(2x+1)^2} \qquad \text{(Substitution)}$$

$$225 + x^2 = 4x^2 + 4x + 1 \qquad \text{(Simplify)}$$

$$0 = 3x^2 + 4x - 224 \qquad \text{(Simplify)}$$

A quadratic equation is of the form $ax^2 + bx + c = 0$. To solve for x, use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

substituting the values of the coefficients a, b and c.

Solution(continued):

With the equation

$$3x^2 + 4x - 224 = 0$$

We have $a = 3, b = 4,$ and $c = -224.$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-224)}}{2(3)}$$

$$x = \frac{-4 \pm \sqrt{16 + 2688}}{6}$$

$$x = \frac{-4 \pm \sqrt{2704}}{6}$$

$$x = \frac{-4 \pm 52}{6}$$

Solution(continued):

$$x = \frac{-4 + 52}{6}, \; x = \frac{-4 - 52}{6}$$

$$x = \frac{48}{6}, \; x = \frac{-56}{6}$$

$$x = 8, \; x = \frac{-28}{3}$$

since x represents a distance, it cannot be negative so the acceptable answer is 8. The missing leg is 8 cm and the hypotenuse is $2(8) + 1 = 17$ cm.

The missing leg is 8 cm, and its hypotenuse is 17 cm.

Start at ELAC, Go Anywhere